B-Tree Analysis

- Let $d = \left\lceil \frac{n}{2} \right\rceil$ and $e = \left\lceil \frac{n-1}{2} \right\rceil$
- Suppose that a B-Tree consists of $m \geq 1$ levels, where the root is level 0.
- Then level *i*, where $1 \leq i \leq m-1$, must contain at least d^{i-1} nodes, otherwise some node at level *i-1* or less would contain fewer than *d* children, which is not possible, or the root would have fewer than one child, which is also not possible.
- The total number of leaf nodes is therefore at least d^{m-2}.
- The total number of search key values *k* is therefore at least ed^{m-2}, i.e., $k \geq ed^{m-2}$
- Dividing by *e* and taking the log of both sides gives $m \leq 2 + \log_d\left(\frac{k}{e}\right)$
- Thus, the height of the tree *m* is logarithmic in *k*, which is the number of search key values in the tree.
- Although this isn't constant time, note that *d* is typically quite large and, consequently, *m* is quite small.