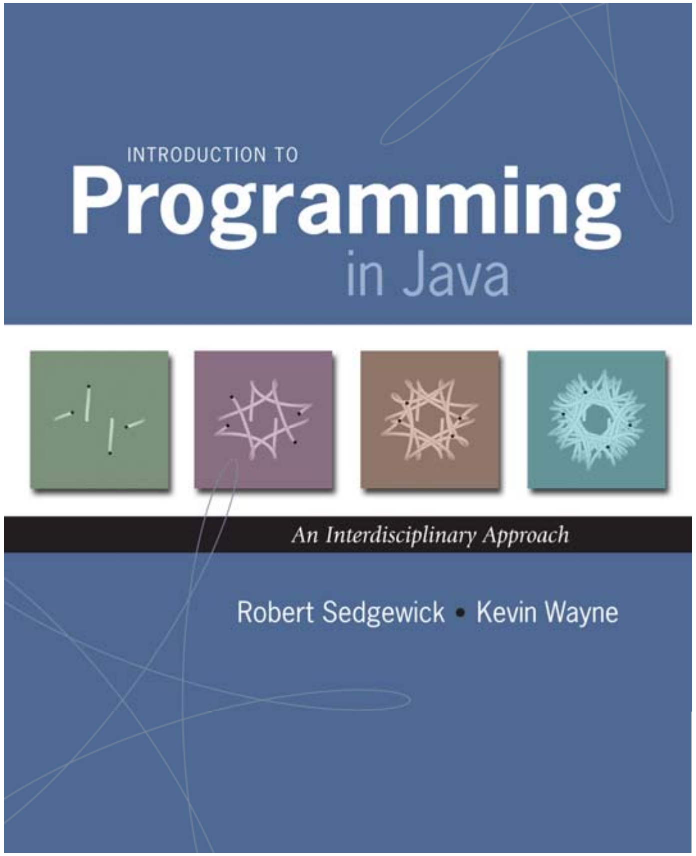
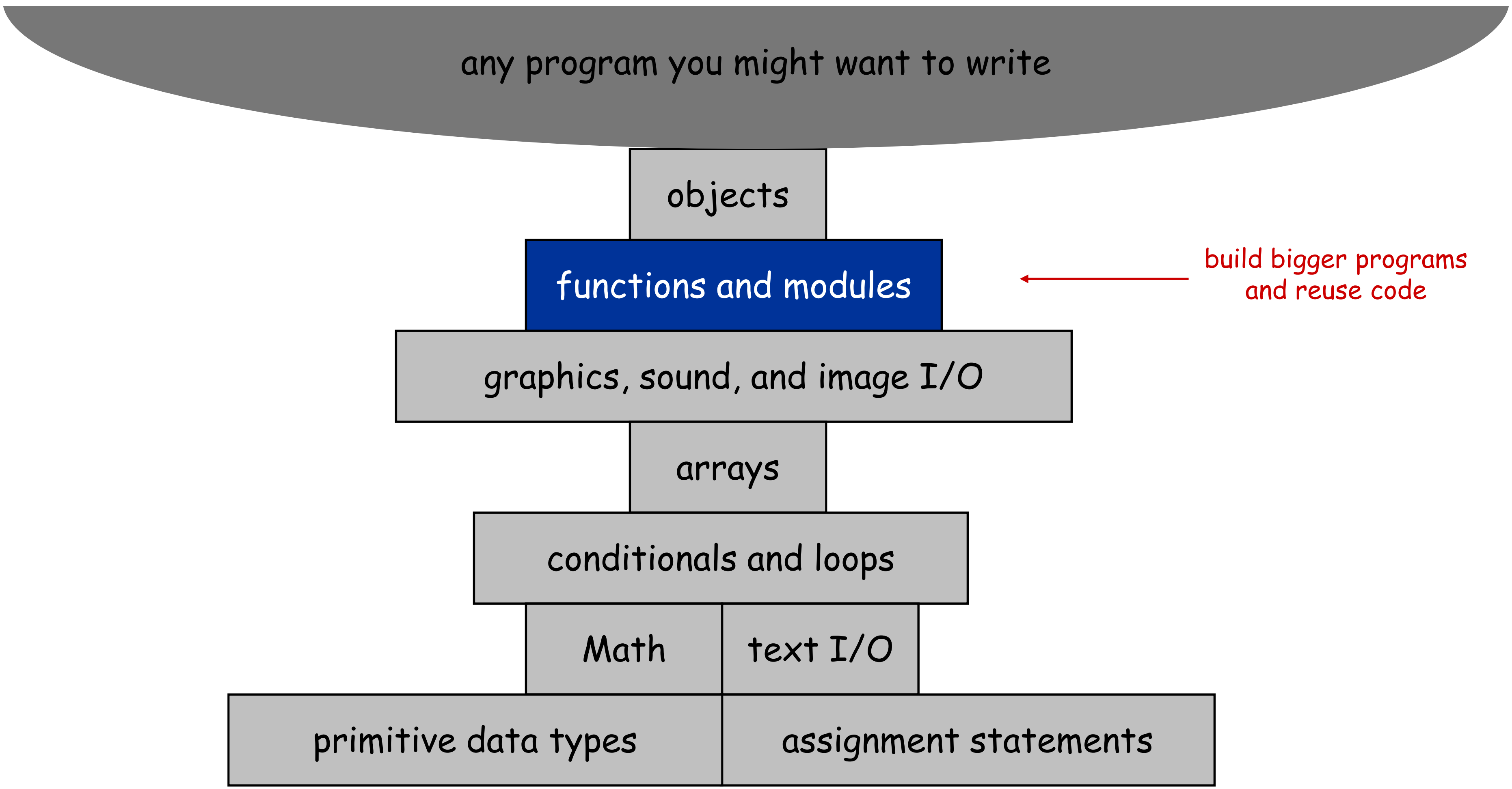
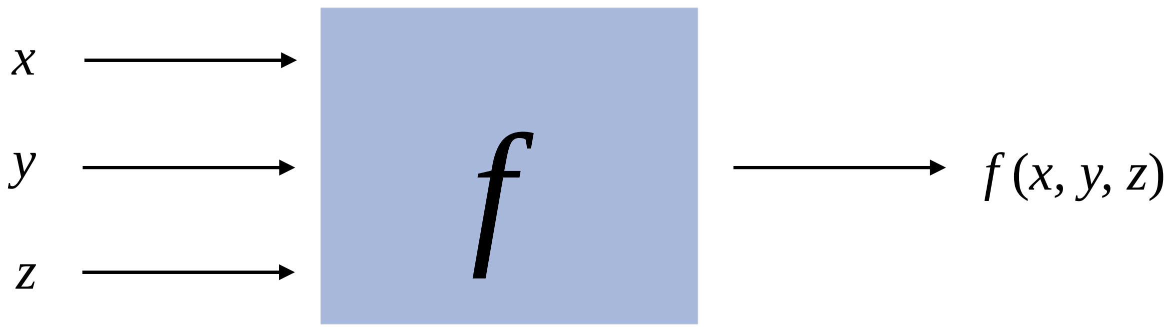
# 2.1 Functions



# A Foundation for Programming



# 2.1 Functions



# Functions (Static Methods)

Java function.

- Takes zero or more input arguments.
- Returns one output value.

Applications.

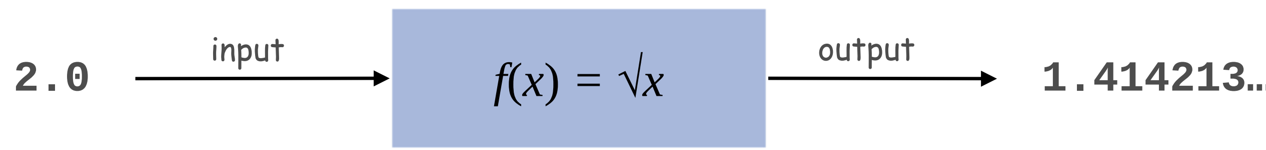
- Scientists use mathematical functions to calculate formulas.
- Programmers use functions to build modular programs.
- You use functions for both.

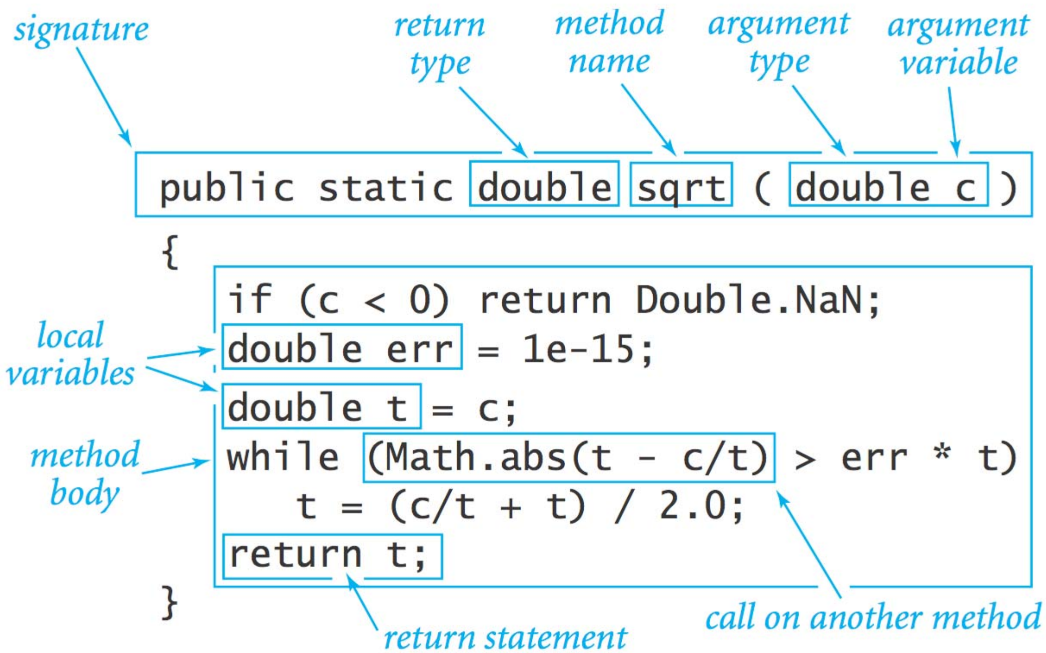
Examples.

- Built-in functions: `Math.random()`, `Math.abs()`, `Integer.parseInt()`.
- Our I/O libraries: `StdIn.readInt()`, `StdDraw.line()`, `StdAudio.play()`.
- User-defined functions: `main()`.

# Anatomy of a Java Function

Java functions. Easy to write your own.




```
public static double sqrt ( double c )
{
   if (c < 0) return Double.NaN;
   double err = 1e-15;
   double t = c;
   while (Math.abs(t - c/t) > err * t)
      t = (c/t + t) / 2.0;
   return t;
}
```

# Scope

Scope (of a name). The code that can refer to that name.

Ex. A variable's scope is code following the declaration in the block.

```
public class Newton {
   public static double sqrt(double c) {
      double epsilon = 1e-15;
      if (c < 0) return Double.NaN;
      double t = c;
      while (Math.abs(t - c/t) > epsilon * t)
         t = (c/t + t) / 2.0;
      return t;
   }

   public static void main(String[] args) {
      double[] a = new double[args.length];
      for (int i = 0; i < args.length; i++)
         a[i] = Double.parseDouble(args[i]);
      for (int i = 0; i < a.length; i++) {
         double x = sqrt(a[i]);
         StdOut.println(x);
      }
   }
}
```

scope of `c`

scope of `epsilon`

scope of `t`

two different variables with the same name `i`

scope of `a`

Best practice: declare variables to limit their scope.

# Flow of Control

**Key point.** Functions provide a new way to control the flow of execution.

```
public class Newton
{
   public static double sqrt(double c)
   {
      if (c < 0) return Double.NaN;
      double err = 1e-15;
      double t = c;
      while (Math.abs(t - c/t) > err * t)
         t = (c/t + t) / 2.0;
      return t;
   }

   public static void main(String[] args)
   {
      int N = args.length;
      double[] a = new double[N];
      for (int i = 0; i < N; i++)
         a[i] = Double.parseDouble(args[i]);
      for (int i = 0; i < N; i++)
      {
         double x = sqrt(a[i]);

         StdOut.println(x);
      }
   }
}
```

# Flow of Control

Key point. Functions provide a new way to control the flow of execution.

Summary of what happens when a function is called:

- Control transfers to the function code.
- Argument variables are assigned the values given in the call.
- Function code is executed.
- Return value is assigned in place of the function name in calling code.
- Control transfers back to the calling code.

Note. This is known as "pass by value."

# Function Challenge 1a

Q. What happens when you compile and run the following code?

```
public class Cubes1 {

   public static int cube(int i) {
      int j = i * i * i;
      return j;
   }

   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

```
% javac Cubes1.java
% java Cubes1 6
1 1
2 8
3 27
4 64
5 125
6 216
```

# Function Challenge 1b

Q. What happens when you compile and run the following code?

```
public class Cubes2 {
   public static int cube(int i) {
      int i = i * i * i;
      return i;
   }
   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

# Function Challenge 1c

Q. What happens when you compile and run the following code?

```
public class Cubes3 {
   public static int cube(int i) {
      i = i * i * i;
   }

   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

# Function Challenge 1d

**Q.** What happens when you compile and run the following code?

```
public class Cubes4 {

   public static int cube(int i) {
      i = i * i * i;
      return i;
   }

   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

# Function Challenge 1e

Q. What happens when you compile and run the following code?

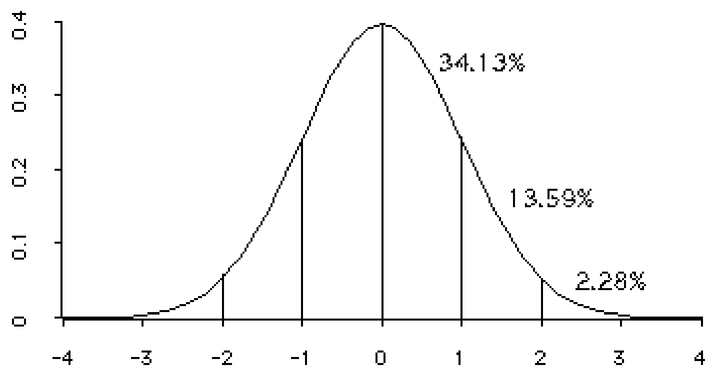
```
public class Cubes5 {
   public static int cube(int i) {
      return i * i * i;
   }

   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      for (int i = 1; i <= N; i++)
         StdOut.println(i + " " + cube(i));
   }
}
```

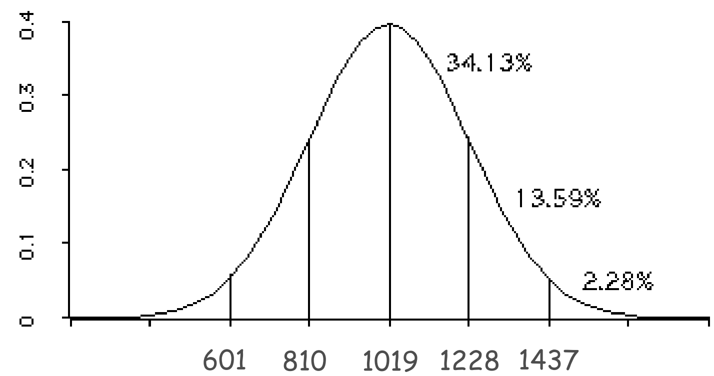
# Gaussian Distribution

# Gaussian Distribution

Standard Gaussian distribution.
- "Bell curve."
- Basis of most statistical analysis in social and physical sciences.

Ex. 2000 SAT scores follow a Gaussian distribution with mean $\mu$ = 1019, stddev $\sigma$ = 209.




$$\phi(x) = \frac{1}{\sqrt{2\pi}} \; e^{-x^2/2}$$

$$\phi(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-(x-\mu)^2 \,/\, 2\sigma^2}$$
$$= \phi\left(\frac{x-\mu}{\sigma}\right) / \; \sigma$$

# Java Function for $\phi(x)$

Mathematical functions. Use built-in functions when possible; build your own when not available.

```
public class Gaussian {

   public static double phi(double x) {
      return Math.exp(-x*x / 2) / Math.sqrt(2 * Math.PI);
   }

   public static double phi(double x, double mu, double sigma) {
      return phi((x - mu) / sigma) / sigma;
   }
}
```

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \; e^{-x^2/2}$$

$$\phi(x, \mu, \sigma) = \phi\left(\frac{x-\mu}{\sigma}\right) / \sigma$$

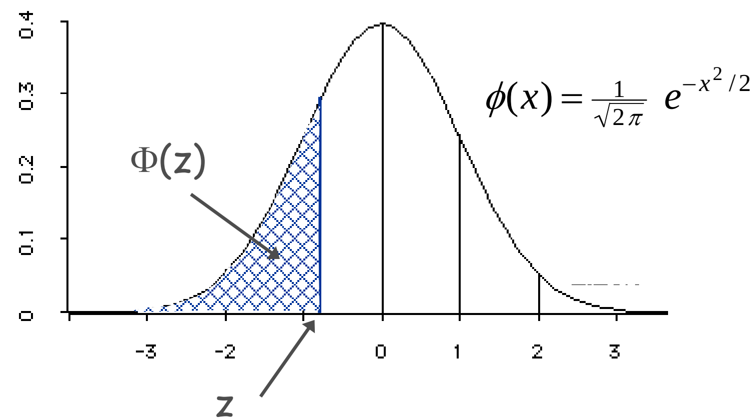
Overloading. Functions with different signatures are different.
Multiple arguments. Functions can take any number of arguments.
Calling other functions. Functions can call other functions.

library or user-defined

# Gaussian Cumulative Distribution Function

Goal. Compute Gaussian cdf $\Phi(z)$.

Challenge. No "closed form" expression and not in Java library.




Taylor series

$$\begin{aligned}\Phi(z) &= \int_{-\infty}^{z} \phi(x)\,dx \\ &= \frac{1}{2} + \phi(z)\left(z + \frac{z^3}{3} + \frac{z^5}{3\cdot 5} + \frac{z^7}{3\cdot 5\cdot 7} + \ldots\right)\end{aligned}$$

Bottom line. 1,000 years of mathematical formulas at your fingertips.

# Java function for $\Phi(z)$

```
public class Gaussian {
   public static double phi(double x)
      // as before

   public static double Phi(double z) {
      if (z < -8.0) return 0.0;
      if (z >  8.0) return 1.0;
      double sum = 0.0, term = z;
      for (int i = 3; sum + term != sum; i += 2) {
         sum  = sum + term;
         term = term * z * z / i;
      }
      return 0.5 + sum * phi(z);
   }

   public static double Phi(double z, double mu, double sigma) {
      return Phi((z - mu) / sigma);
   }
}
```

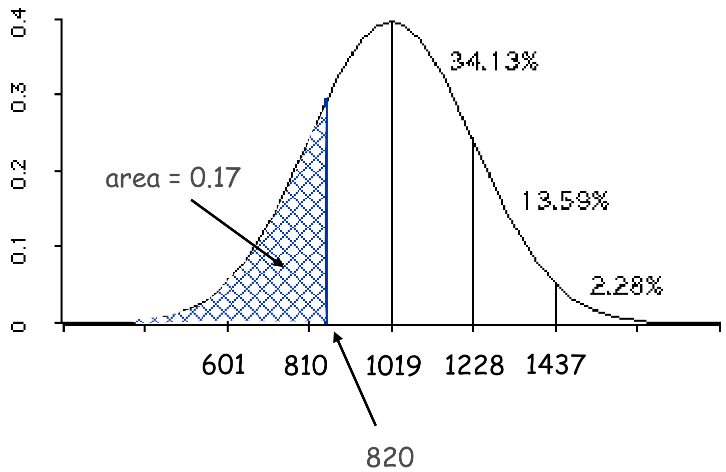
accurate with absolute error less than $8 * 10^{-16}$

$$\Phi(z,\ \mu,\ \sigma) \ = \ \int_{-\infty}^{z} \phi(z, \mu, \sigma) \ = \ \Phi((z-\mu)\ /\ \sigma)$$

# SAT Scores

Q. NCAA requires at least 820 for Division I athletes.
What fraction of test takers in 2000 do not qualify?

A. $\Phi(820, \mu, \sigma) \approx 0.17051$. [approximately 17%]




```
double fraction = Gaussian.Phi(820, 1019, 209);
```

# Gaussian Distribution

Q. Why relevant in mathematics?

A. Central limit theorem: under very general conditions, average of a set of variables tends to the Gaussian distribution.

Q. Why relevant in the sciences?

A. Models a wide range of natural phenomena and random processes.

- Weights of humans, heights of trees in a forest.
- SAT scores, investment returns.

Caveat.

> Everybody believes in the exponential law of errors: the experimenters, because they think it can be proved by mathematics; and the mathematicians, because they believe it has been established by observation. - M. Lippman in a letter to H. Poincaré

# Building Functions

Functions enable you to build a new layer of abstraction.

- Takes you beyond pre-packaged libraries.
- You build the tools you need: `Gaussian.phi()`, …

Process.

- Step 1: identify a useful feature.
- Step 2: implement it.
- Step 3: use it.

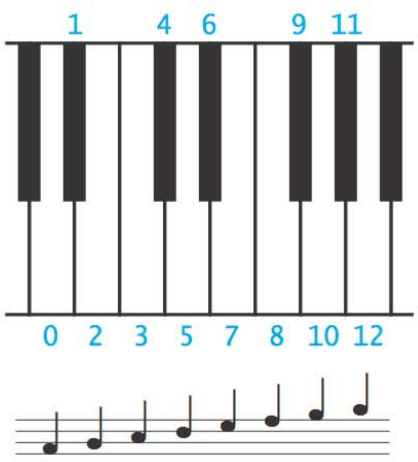
- Step 3': re-use it in any of your programs.

# Digital Audio

# Crash Course in Sound

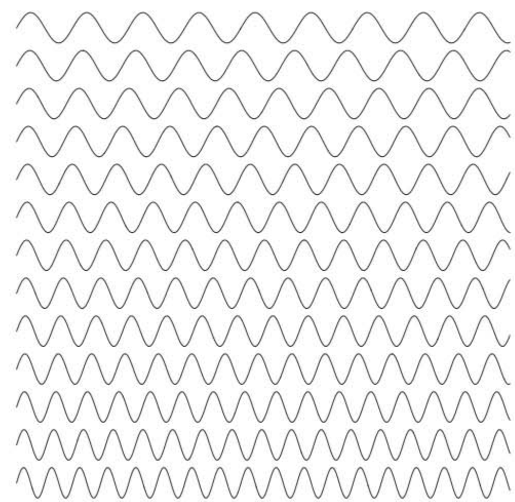
**Sound.** Perception of the vibration of molecules in our eardrums.

**Concert A.** Sine wave, scaled to oscillated at 440Hz.
**Other notes.** 12 notes on chromatic scale, divided logarithmically.




| *note* | *i* | *frequency* |
|---|---|---|
| A | 0 | 440.00 |
| A♯ *or* B♭ | 1 | 466.16 |
| B | 2 | 493.88 |
| C | 3 | 523.25 |
| C♯ *or* D♭ | 4 | 554.37 |
| D | 5 | 587.33 |
| D♯ *or* E♭ | 6 | 622.25 |
| E | 7 | 659.26 |
| F | 8 | 698.46 |
| F♯ *or* G♭ | 9 | 739.99 |
| G | 10 | 783.99 |
| G♯ *or* A♭ | 11 | 830.61 |
| A | 12 | 880.00 |

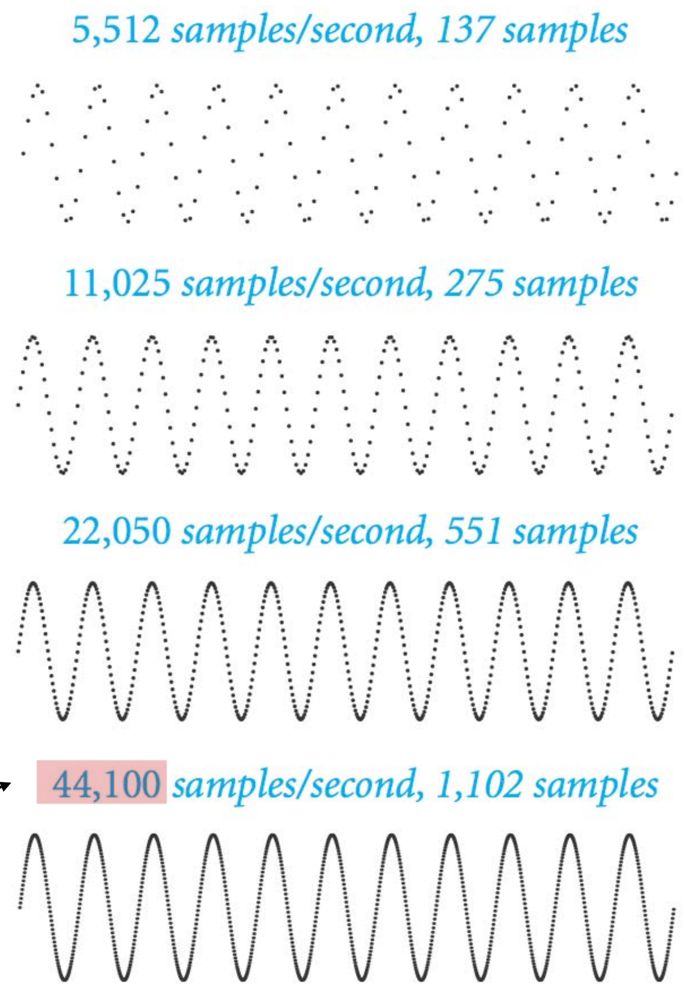
$440 \times 2^{i/12}$



*Notes, numbers, and waves*

# Digital Audio

**Sampling.** Represent curve by sampling it at regular intervals.




audio CD

$$y(i) \;=\; \sin\left(\frac{2\,\pi \cdot i \cdot 440}{44{,}100}\right)$$

# Musical Tone Function

Musical tone. Create a music tone of a given frequency and duration.

```
public static double[] tone(double hz, double seconds) {
   int SAMPLE_RATE = 44100;
   int N = (int) (seconds * SAMPLE_RATE);
   double[] a = new double[N+1];
   for (int i = 0; i <= N; i++) {
      a[i] = Math.sin(2 * Math.PI * i * hz / SAMPLE_RATE);
   }
   return a;
}
```

$$y(i) = \sin\left(\frac{2\pi \cdot i \cdot hz}{44{,}100}\right)$$

Remark. Can use arrays as function return value and/or argument.

# Digital Audio in Java

Standard audio. Library for playing digital audio.

| public class StdAudio | |
|---|---|
| void play(String file) | *play the given .wav file* |
| void play(double[] a) | *play the given sound wave* |
| void play(double x) | *play sample for 1/44100 second* |
| void save(String file, double[] a) | *save to a .wav file* |
| void double[] read(String file) | *read from a .wav file* |

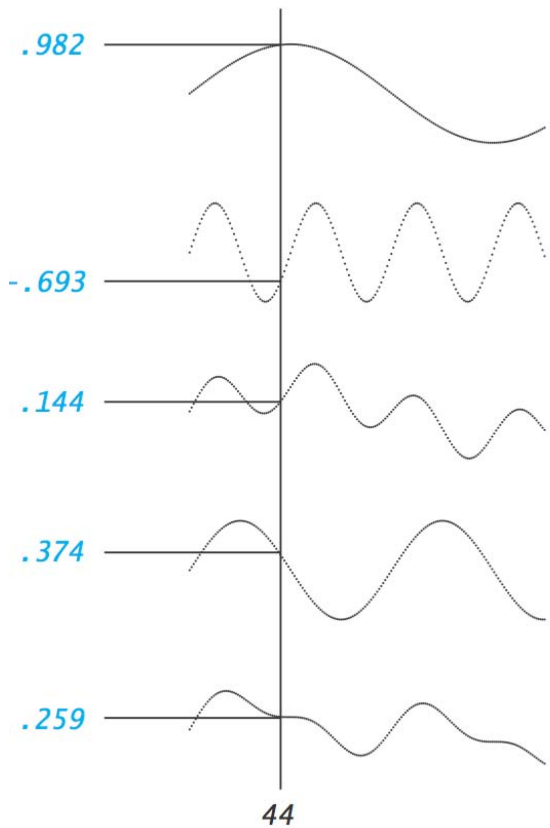
Concert A. Play concert A for `1.5` seconds using `StdAudio`.

```
double[] a = tone(440, 1.5);
StdAudio.play(a);
```

# Harmonics

**Concert A with harmonics.** Obtain richer sound by adding tones one octave above and below concert A.

880 Hz (above) 220 Hz (below) 440 Hz (concert A)




```
lo = tone(220, .0041);
lo[44] = .982

hi = tone(880, .0041);
hi[44] = -.693

h = sum(hi, lo, .5, .5);
h[44] = .5*lo[44]+.5*hi[44];
      = .5*.982 - .5*.693 = .144

A = tone(440, .0041);
A[44] = .374

sum(A, h, .5, .5);
A[44] + h[44] = .5*.144 + .5*.374
              = .259
```

# Harmonics

```
public class PlayThatTune {

   // return weighted sum of two arrays
   public static double[] sum(double[] a, double[] b, double awt, double bwt) {
      double[] c = new double[a.length];
      for (int i = 0; i < a.length; i++)
         c[i] = a[i]*awt + b[i]*bwt;
      return c;
   }

   // return a note of given pitch and duration
   public static double[] note(int pitch, double duration) {
      double hz = 440.0 * Math.pow(2, pitch / 12.0);
      double[] a  = tone(1.0 * hz, duration);
      double[] hi = tone(2.0 * hz, duration);
      double[] lo = tone(0.5 * hz, duration);
      double[] h  = sum(hi, lo, .5, .5);
      return sum(a, h, .5, .5);
   }

   public static double[] tone(double hz, double t)
      // see previous slide

   public static void main(String[] args)
      // see next slide
}
```

# Harmonics

Play that tune. Read in pitches and durations from standard input, and play using standard audio.

```java
public static void main(String[] args) {
   while (!StdIn.isEmpty()) {
      int pitch = StdIn.readInt();
      double duration = StdIn.readDouble();
      double[] a = note(pitch, duration);
      StdAudio.play(a);
   }
}
```

```
% more elise.txt
7 .125
6 .125
7 .125
6 .125
7 .125
2 .125
5 .125
3 .125
0 .25
```

```
% java PlayThatTune < elise.txt
```

```
public class PlayThatTune
{
   public static double[] sum(double[] a, double[] b,
                              double awt, double bwt)
   {
      double[] c = new double[a.length];
      for (int i = 0; i < a.length; i++)
         c[i] = a[i]*awt + b[i]*bwt;
      return c;
   }

   public static double[] tone(double hz, double t)
   {
      int sps = 44100;
      int N = (int) (sps * t);
      double[] a = new double[N+1];
      for (int i = 0; i <= N; i++)
         a[i] = Math.sin(2 * Math.PI * i * hz / sps);
      return a;
   }

   public static double[] note(int pitch, double t)
   {
      double hz = 440.0 * Math.pow(2, pitch / 12.0);
      double[] a  = tone(hz, t);
      double[] hi = tone(2*hz, t);
      double[] lo = tone(hz/2, t);
      double[] h  = sum(hi, lo, .5, .5);
      return sum(a, h, .5, .5);
   }

   public static void main(String[] args)
   {
      while (!StdIn.isEmpty())
      {
         int pitch = StdIn.readInt();
         double duration = StdIn.readDouble();
         double[] a = note(pitch, duration);
         StdAudio.play(a);
      }
   }
}
```

# Extra Slides

# Functions

| | |
|---|---|
| *absolute value of an* `int` *value* | `public static int abs(int x)`<br>`{`<br>`   if (x < 0) return -x;`<br>`   else       return  x;`<br>`}` |
| *absolute value of a* `double` *value* | `public static double abs(double x)`<br>`{`<br>`   if (x < 0.0) return -x;`<br>`   else         return  x;`<br>`}` |
| *primality test* | `public static boolean isPrime(int N)`<br>`{`<br>`   if (N < 2) return false;`<br>`   for (int i = 2; i <= N/i; i++)`<br>`      if (N % i == 0) return false;`<br>`   return true;`<br>`}` |
| *hypotenuse of a right triangle* | `public static double hypotenuse(double a, double b)`<br>`{  return Math.sqrt(a*a + b*b);  }` |

overloading

multiple arguments