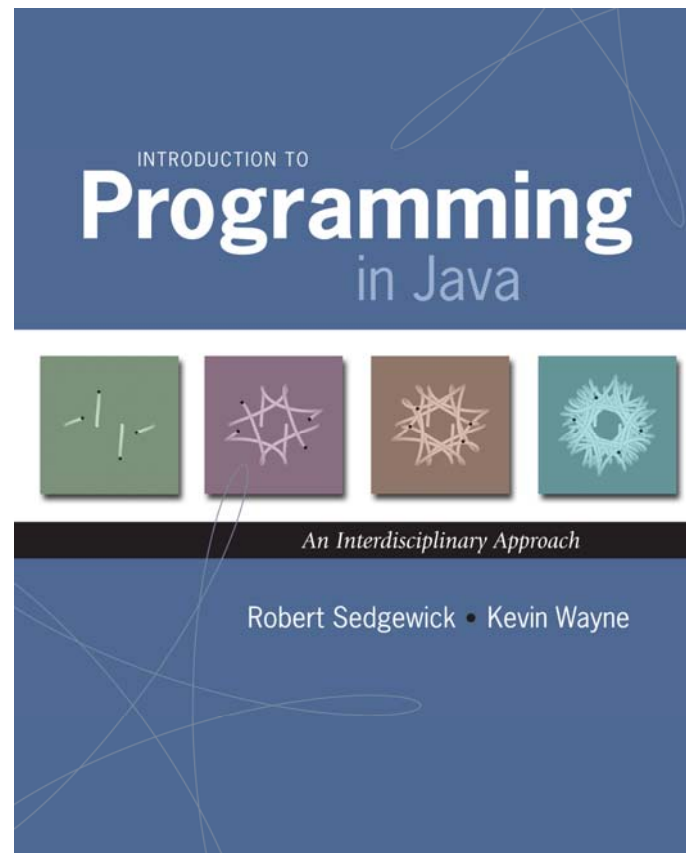
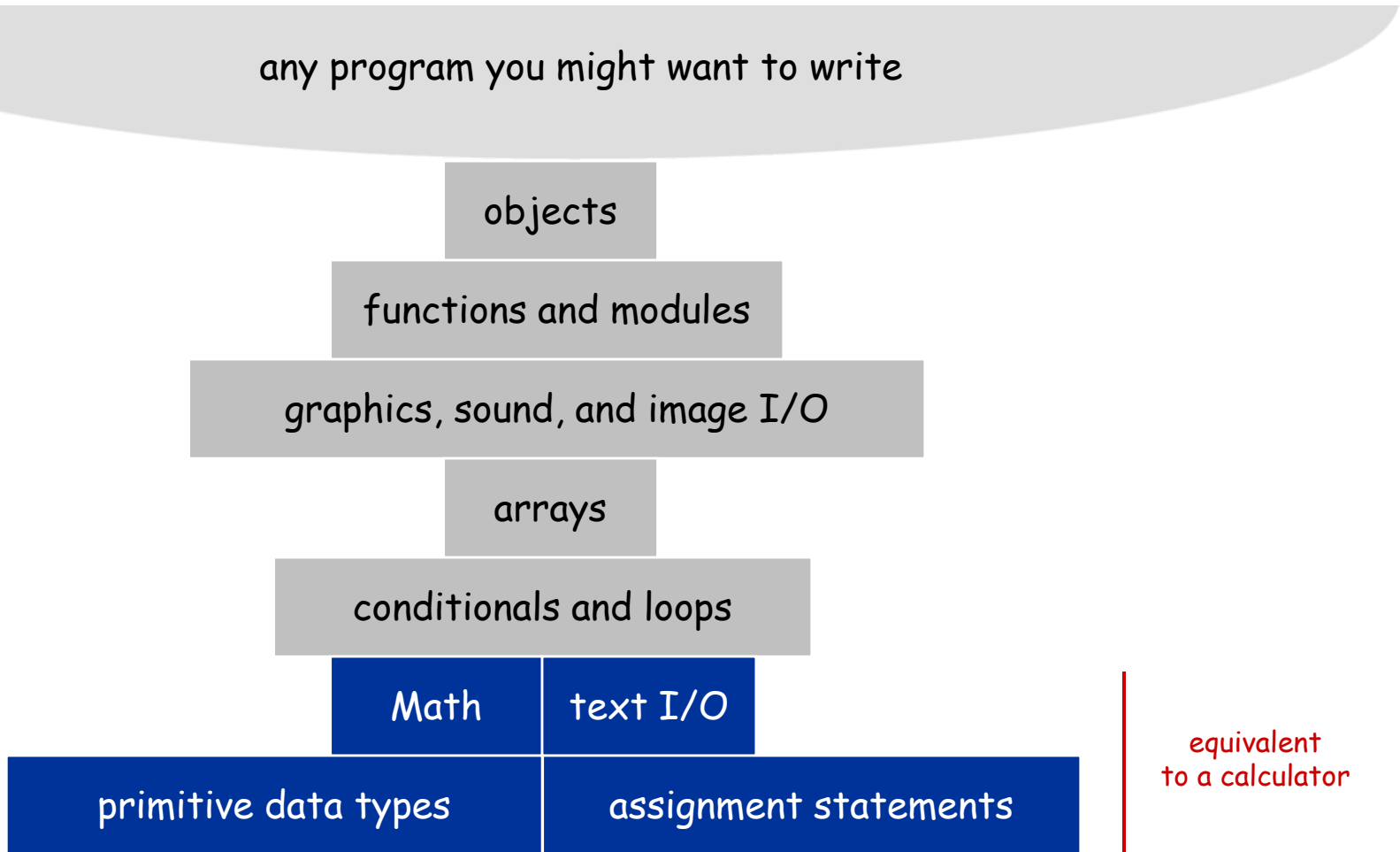


1.3 Conditionals and Loops



A Foundation for Programming



A Foundation for Programming

any program you might want to write

objects

functions and modules

graphics, sound, and image I/O

arrays

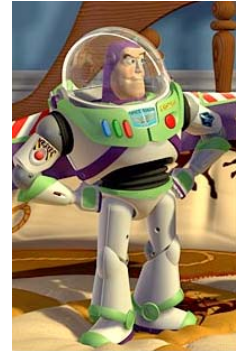
conditionals and loops

Math

text I/O

primitive data types

assignment statements



to infinity
and beyond!

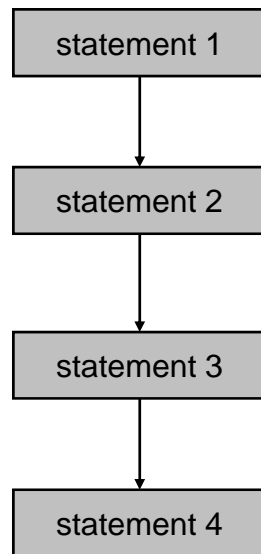




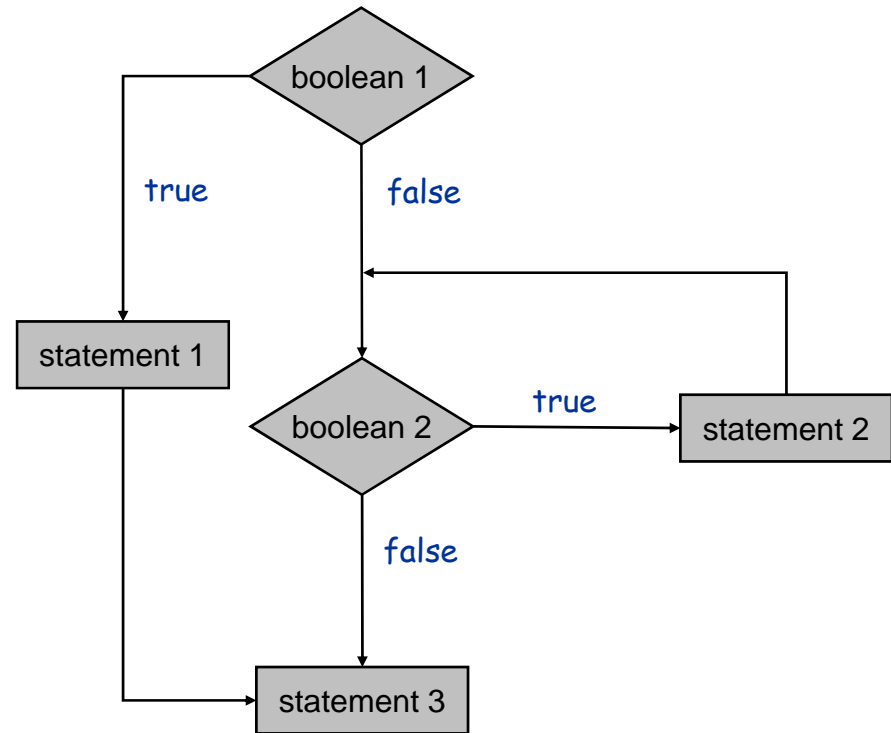
Control Flow

Control flow.

- Sequence of statements that are actually executed in a program.
- Conditionals and loops: enable us to choreograph control flow.



straight-line control flow



control flow with conditionals and loops

Conditionals

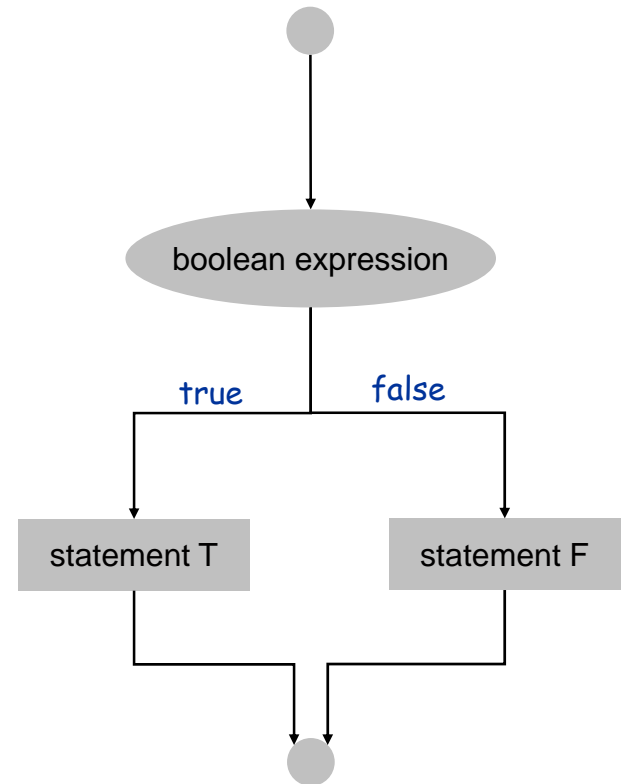
If Statement

The `if` statement. A common branching structure.

- Check `boolean` condition.
- If `true`, execute some statements.
- If `false`, execute other statements.

```
if (boolean expression) {  
    statement T;  
}  
else {  
    statement F;  
}
```

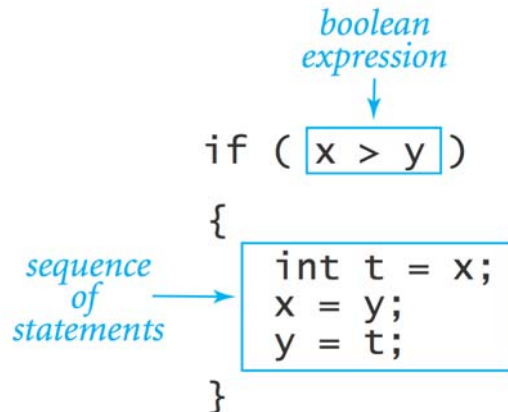
← can be any sequence
of statements



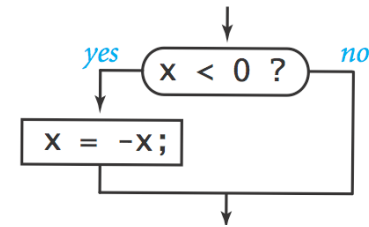
If Statement

The `if` statement. A common branching structure.

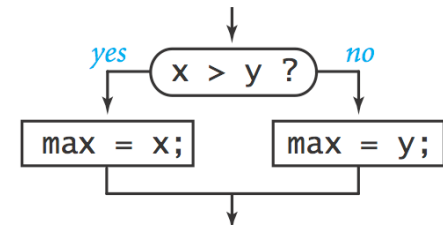
- Check `boolean` condition.
- If `true`, execute some statements.
- If `false`, execute other statements.



```
if (x < 0) x = -x;
```



```
if (x > y) max = x;  
else      max = y;
```



If Statement

Ex. Take different action depending on value of variable.

```
public class Flip {  
    public static void main(String[] args) {  
        if (Math.random() < 0.5) System.out.println("Heads");  
        else System.out.println("Tails");  
    }  
}
```



If Statement Examples

<i>absolute value</i>	<pre>if (x < 0) x = -x;</pre>
<i>put x and y into sorted order</i>	<pre>if (x > y) { int t = x; y = x; x = t; }</pre>
<i>maximum of x and y</i>	<pre>if (x > y) max = x; else max = y;</pre>
<i>error check for division operation</i>	<pre>if (den == 0) System.out.println("Division by zero"); else System.out.println("Quotient = " + num/den);</pre>
<i>error check for quadratic formula</i>	<pre>double discriminant = b*b - 4.0*c; if (discriminant < 0.0) { System.out.println("No real roots"); } else { System.out.println((-b + Math.sqrt(discriminant))/2.0); System.out.println((-b - Math.sqrt(discriminant))/2.0); }</pre>

The While Loop

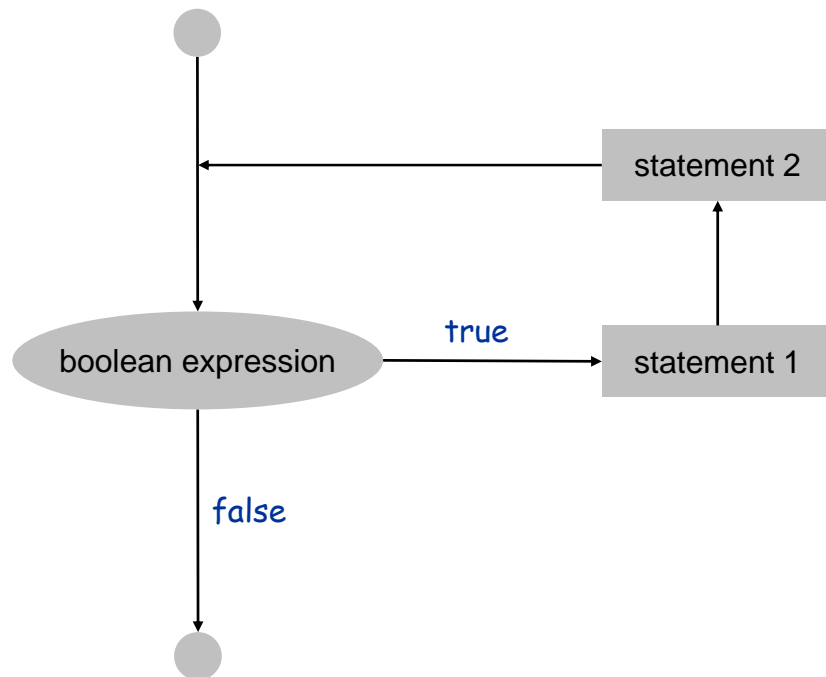
While Loop

The **while** loop. A common repetition structure.

- Check a boolean expression.
- Execute a sequence of statements.
- Repeat.

```
while (boolean expression) {  
    statement 1;  
    statement 2;  
}
```

Diagram labels:
- "loop continuation condition" points to the `boolean expression` in the code.
- "loop body" points to the statements inside the curly braces.





While Loops: Powers of Two

Ex. Print first n powers of 2.

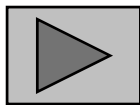
- Increment i from 1 to n .
- Double v each time.

```
int i = 0;
int v = 1;
while (i <= N) {
    System.out.println(v);
    i = i + 1;
    v = 2 * v;
}
```

i	v	i <= N
0	1	true
1	2	true
2	4	true
3	8	true
4	16	true
5	32	true
6	64	true
7	128	false

```
1
2
4
8
16
32
64
```

$n = 6$



Click for demo

Powers of Two

```
public class PowersOfTwo {  
    public static void main(String[] args) {  
  
        // last power of two to print  
        int N = Integer.parseInt(args[0]);  
  
        int i = 0; // loop control counter  
        int v = 1; // current power of two  
        while (i <= N) {  
            System.out.println(v);  
            i = i + 1;  
            v = 2 * v;  
        }  
    }  
}
```

print ith power of two

```
% java PowersOfTwo 4  
1  
2  
4  
8  
  
% java PowersOfTwo 6  
1  
2  
4  
8  
16  
32  
64
```



While Loop Challenge

Q. Anything wrong with the following code for printing powers of 2?

```
int i = 0;
int v = 1;
while (i <= N)
    System.out.println(v);
    i = i + 1;
    v = 2 * v;
```



While Loop Challenge

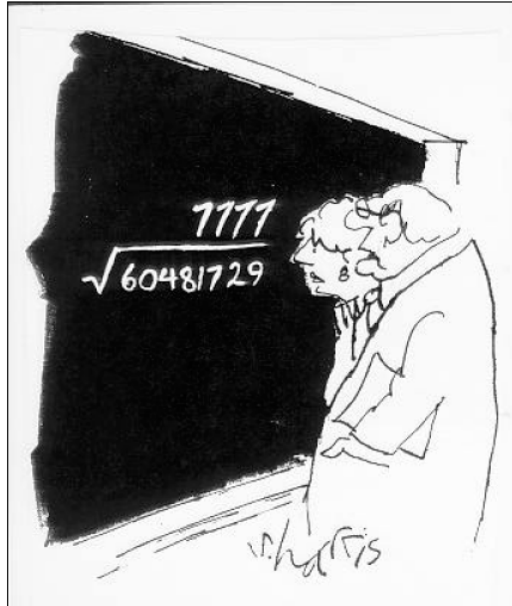
Q. Anything wrong with the following code for printing powers of 2?

```
int i = 0;
int v = 1;
while (i <= N)
    System.out.println(v);
    i = i + 1;
    v = 2 * v;
```

A. Need curly braces around statements in while loop; otherwise it enters an infinite loop, printing 1s.

Moment of panic. How to stop infinite loop?

A Wonderful Square Root



"A wonderful square root. Let's hope it can be used for the good of mankind."

Copyright 2004, Sidney Harris, <http://www.sciencecartoonsplus.com>

```
% java Sqrt 60481729  
7777.0
```




While Loops: Square Root

Q. How might we implement `Math.sqrt()` ?

A. To compute the square root of c :

- Initialize $t_0 = c$.
- **Repeat until** $t_i = c / t_i$, up to desired precision:
set t_{i+1} to be the average of t_i and c / t_i .

$$\begin{aligned} t_0 &= 2.0 \\ t_1 &= \frac{1}{2} \left(t_0 + \frac{2}{t_0} \right) = 1.5 \\ t_2 &= \frac{1}{2} \left(t_1 + \frac{2}{t_1} \right) = 1.4166666666666665 \\ t_3 &= \frac{1}{2} \left(t_2 + \frac{2}{t_2} \right) = 1.4142156862745097 \\ t_4 &= \frac{1}{2} \left(t_3 + \frac{2}{t_3} \right) = 1.4142135623746899 \\ t_5 &= \frac{1}{2} \left(t_4 + \frac{2}{t_4} \right) = 1.414213562373095 \end{aligned}$$

computing the square root of 2

While Loops: Square Root

Q. How might we implement `Math.sqrt()` ?

A. To compute the square root of c :

- Initialize $t_0 = c$.
- **Repeat until** $t_i = c / t_i$, up to desired precision:
set t_{i+1} to be the average of t_i and c / t_i .

```
public class Sqrt {  
    public static void main(String[] args) {  
        double EPS = 1E-15;  
        double c = Double.parseDouble(args[0]);  
        double t = c;  
        while (Math.abs(t - c/t) > t*EPS) {  
            t = (c/t + t) / 2.0;  
        }  
        System.out.println(t);  
    }  
}
```

error tolerance

```
% java Sqrt 2.0  
1.414213562373095
```

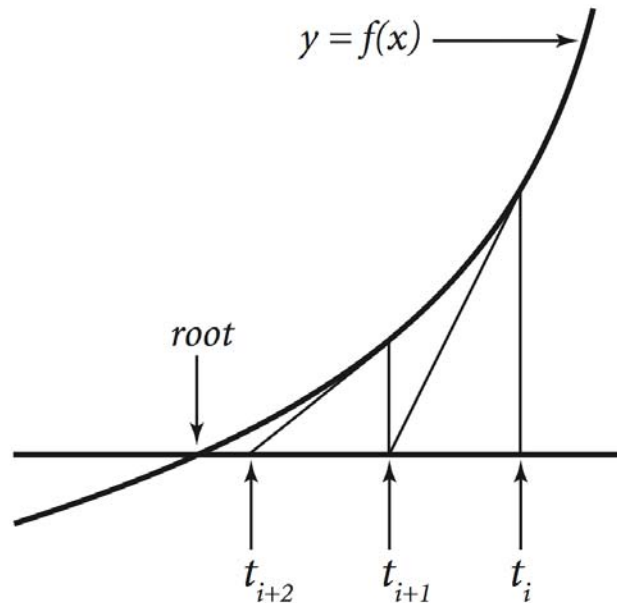
15 decimal digits of accuracy in 5 iterations

Newton-Raphson Method

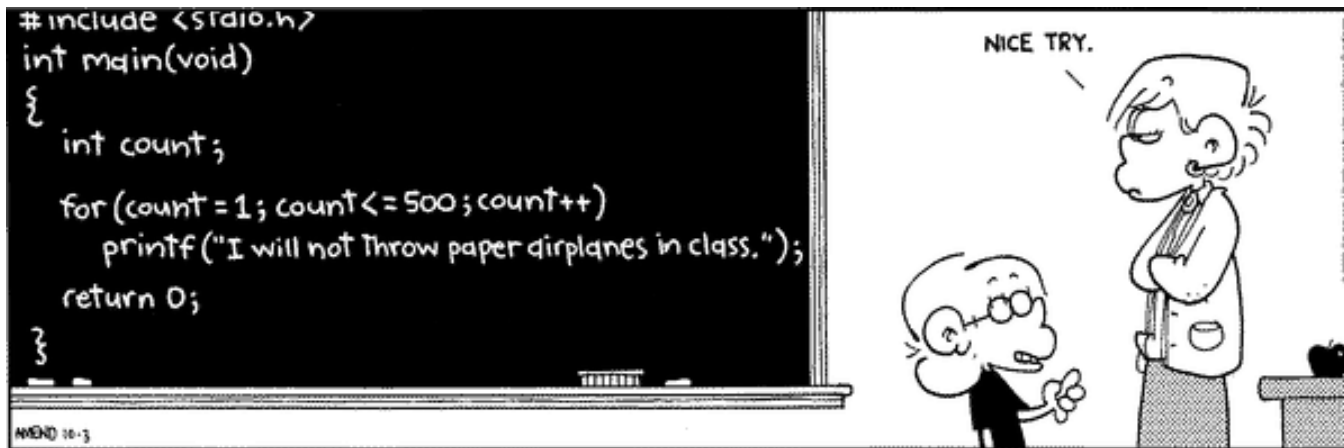
Square root method explained.

- Goal: find root of function $f(x)$.
- Start with estimate t_0 .
- Draw line tangent to curve at $x = t_i$.
- Set t_{i+1} to be x -coordinate where line hits x -axis.
- Repeat until desired precision.

$f(x) = x^2 - c$ to compute \sqrt{c}



The For Loop



Copyright 2004, FoxTrot by Bill Amend
www.ucomics.com/foxtrot/2003/10/03

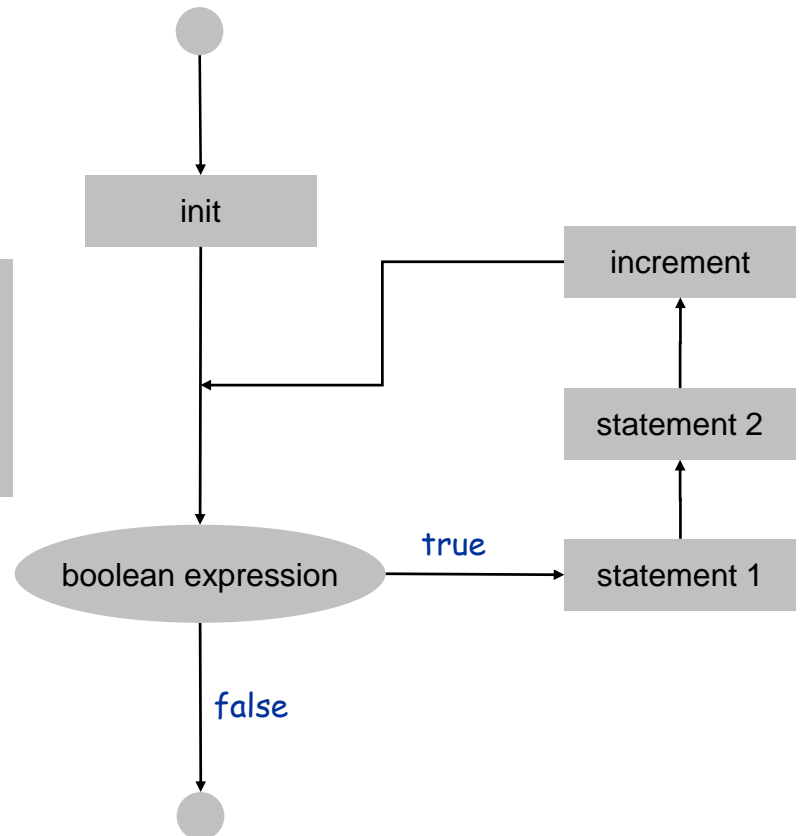
For Loops

The `for` loop. Another common repetition structure.

- Execute initialization statement.
- Check boolean expression.
- Execute sequence of statements.
- Execute increment statement.
- Repeat.

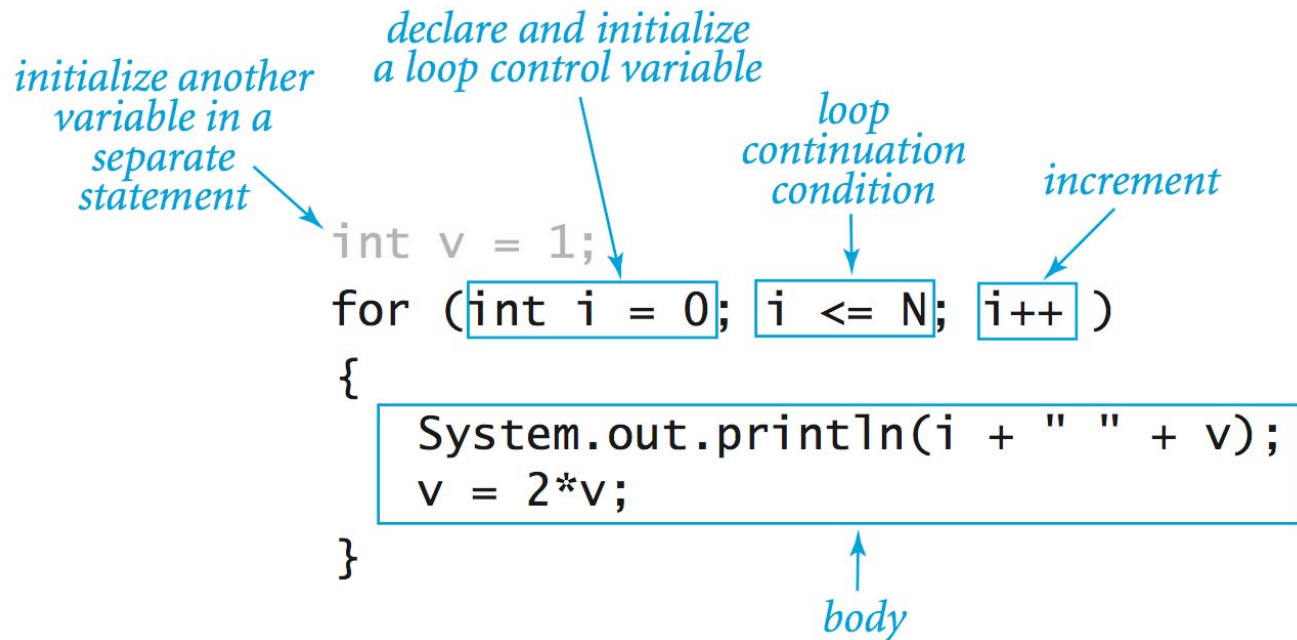
```
for (init; boolean expression; increment) {  
    statement 1;  
    statement 2;  
}
```

Annotations:
- `boolean expression`: loop continuation condition
- `statement 1; statement 2;`: body





Anatomy of a For Loop



Q. What does it print?

A.

For Loops: Subdivisions of a Ruler

Create subdivision of a ruler.

- Initialize `ruler` to empty string.
- For each value `i` from 1 to `N`:
sandwich two copies of `ruler` on either side of `i`.

```
public class Ruler {  
    public static void main(String[] args) {  
        int N = Integer.parseInt(args[0]);  
        String ruler = " ";  
        for (int i = 1; i <= N; i++) {  
            ruler = ruler + i + ruler;  
        }  
        System.out.println(ruler);  
    }  
}
```

i	ruler
	" "
1	" 1 "
2	" 1 2 1 "
3	" 1 2 1 3 1 2 1 "



For Loops: Subdivisions of a Ruler

```
% java Ruler 1
1

% java Ruler 2
1 2 1

% java Ruler 3
1 2 1 3 1 2 1

% java Ruler 4
1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

% java Ruler 5
1 2 1 3 1 2 1 4 1 2 1 3 1 2 1 5 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

% java Ruler 100
Exception in thread "main"
java.lang.OutOfMemoryError
```

Observation. Loops can produce a huge amount of output!

Loop Examples

<i>print powers of two</i>	<pre>int v = 1; for (int i = 0; i <= N; i++) { System.out.println(i + " " + v); v = 2*v; }</pre>
<i>print largest power of two less than or equal to N</i>	<pre>int v = 1; while (v <= N/2) v = 2*v; System.out.println(v);</pre>
<i>compute a finite sum (1 + 2 + ... + N)</i>	<pre>int sum = 0; for (int i = 1; i <= N; i++) sum += i; System.out.println(sum);</pre>
<i>compute a finite product (N! = 1 × 2 × ... × N)</i>	<pre>int product = 1; for (int i = 1; i <= N; i++) product *= i; System.out.println(product);</pre>
<i>print a table of function values</i>	<pre>for (int i = 0; i <= N; i++) System.out.println(i + " " + 2*Math.PI*i/N);</pre>
<i>print the ruler function (see Program 1.2.1)</i>	<pre>String ruler = " "; for (int i = 1; i <= N; i++) ruler = ruler + i + ruler; System.out.println(ruler);</pre>

Nesting



Nesting Conditionals and Loops

Conditionals enable you to do one of 2^n sequences of operations with n lines.

```
if (a0 > 0) System.out.print(0);
if (a1 > 0) System.out.print(1);
if (a2 > 0) System.out.print(2);
if (a3 > 0) System.out.print(3);
if (a4 > 0) System.out.print(4);
if (a5 > 0) System.out.print(5);
if (a6 > 0) System.out.print(6);
if (a7 > 0) System.out.print(7);
if (a8 > 0) System.out.print(8);
if (a9 > 0) System.out.print(9);
```

$2^{10} = 1024$ possible results, depending on input

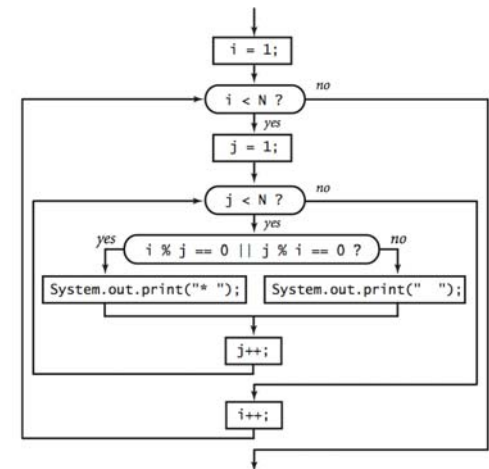
More sophisticated programs.

- Nest conditionals within conditionals.
- Nest loops within loops.
- Nest conditionals within loops within loops.

Loops enable you to do an operation n times using only 2 lines of code.

```
double sum = 0.0;
for (int i = 1; i <= 1024; i++)
    sum = sum + 1.0 / i;
```

computes $1/1 + 1/2 + \dots + 1/1024$





Nested If Statements

Ex. Pay a certain tax rate depending on income level.

Income	Rate
0 - 47,450	22%
47,450 - 114,650	25%
114,650 - 174,700	28%
174,700 - 311,950	33%
311,950 -	35%

5 mutually exclusive alternatives

```
double rate;  
if (income < 47450) rate = 0.22;  
else if (income < 114650) rate = 0.25;  
else if (income < 174700) rate = 0.28;  
else if (income < 311950) rate = 0.33;  
else rate = 0.35;
```

graduated income tax calculation



Nested If Statements

```
if      (income < 47450) rate = 0.22;
else if (income < 114650) rate = 0.25;
else if (income < 174700) rate = 0.28;
else if (income < 311950) rate = 0.33;
else if (income < 311950) rate = 0.35;
```

is shorthand for

```
if (income < 47450) rate = 0.22;
else {
    if (income < 114650) rate = 0.25;
    else {
        if (income < 174700) rate = 0.28;
        else {
            if (income < 311950) rate = 0.33;
            else if (income < 311950) rate = 0.35;
        }
    }
}
```

Be careful when nesting if-else statements (see Q+A p. 75).



Nested If Statement Challenge

Q. Anything wrong with the following for income tax calculation?

Income	Rate
0 - 47,450	22%
47,450 - 114,650	25%
114,650 - 174,700	28%
174,700 - 311,950	33%
311,950 -	35%

```
double rate = 0.35;  
if (income < 47450) rate = 0.22;  
if (income < 114650) rate = 0.25;  
if (income < 174700) rate = 0.28;  
if (income < 311950) rate = 0.33;
```

wrong graduated income tax calculation

Monte Carlo Simulation





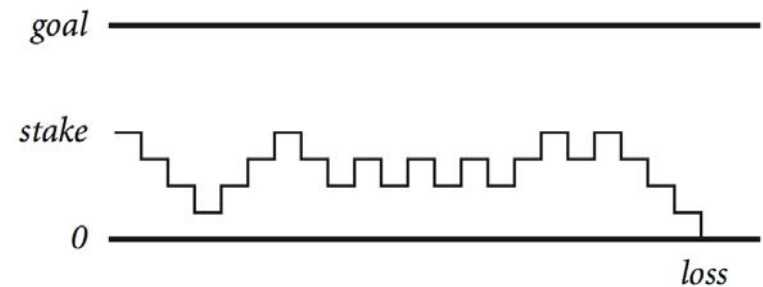
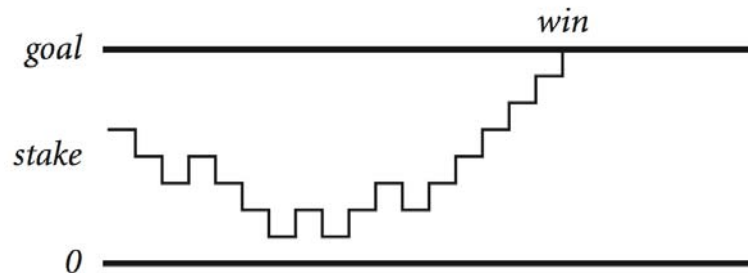
Gambler's Ruin

Gambler's ruin. Gambler starts with \$stake and places \$1 fair bets until going broke or reaching \$goal.

- What are the chances of winning?
- How many bets will it take?

One approach. Monte Carlo simulation.

- Flip digital coins and see what happens.
- Repeat and compute statistics.



Gambler's Ruin

```
public class Gambler {
    public static void main(String[] args) {
        int stake = Integer.parseInt(args[0]);
        int goal = Integer.parseInt(args[1]);
        int trials = Integer.parseInt(args[2]);
        int wins = 0;

        // repeat experiment N times
        for (int i = 0; i < trials; i++) {
            // do one gambler's ruin
            // experiment
            int t = stake;
            while (t > 0 && t < goal) {
                // flip coin and update
                if (Math.random() < 0.5) t++;
                else t--;
            }
            if (t == goal) wins++;
        }
        System.out.println(wins + " wins of " + trials);
    }
}
```



Digression: Simulation and Analysis

```
                stake goal trials
                ↓   ↓   ↓
% java Gambler 5 25 1000
191 wins of 1000

% java Gambler 5 25 1000
203 wins of 1000

% java Gambler 500 2500 1000
197 wins of 1000
```

after a substantial wait...

Fact. Probability of winning = $\text{stake} \div \text{goal}$.

Fact. Expected number of bets = $\text{stake} \times \text{desired gain}$.

Ex. 20% chance of turning \$500 into \$2500,
but expect to make one million \$1 bets.

$$500/2500 = 20\%$$

$$500 * (2500 - 500) = 1 \text{ million}$$

Remark. Both facts can be proved mathematically; for more complex scenarios, computer simulation is often the best plan of attack.

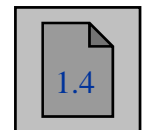


Control Flow Summary

Control flow.

- Sequence of statements that are actually executed in a program.
- Conditionals and loops: enables us to choreograph the control flow.

Control Flow	Description	Examples
Straight-line programs	All statements are executed in the order given.	
Conditionals	Certain statements are executed depending on the values of certain variables.	if if-else
Loops	Certain statements are executed repeatedly until certain conditions are met.	while for do-while





Program Development



Ada Lovelace



Admiral Grace Murray Hopper



95% of Program Development

Debugging. Cyclic process of editing, compiling, and fixing errors.

- Always a logical explanation.
- What would the machine do?
- Explain it to the teddy bear.



You will make many mistakes as you write programs. It's normal.

“As soon as we started programming, we found out to our surprise that it wasn't as easy to get programs right as we had thought. I can remember the exact instant when I realized that a large part of my life from then on was going to be spent in finding mistakes in my own programs. ” — Maurice Wilkes

“ If I had eight hours to chop down a tree, I would spend six hours sharpening an axe. ” — Abraham Lincoln



Debugging Example

Factor. Given an integer $N > 1$, compute its prime factorization.

$$3,757,208 = 2^3 \times 7 \times 13^2 \times 397$$

$$98 = 2 \times 7^2$$

$$17 = 17$$

$$11,111,111,111,111,111 = 2,071,723 \times 5,363,222,357$$

Application. Break RSA cryptosystem (factor 200-digit numbers).



Debugging Example

Factor. Given an integer N , compute its prime factorization.

Brute-force algorithm. For each putative factor $i = 2, 3, 4, \dots$, check if N is a multiple of i , and if so, divide it out.

i	N	<i>output</i>	i	N	<i>output</i>	i	N	<i>output</i>
2	3757208	2 2 2	9	67093		16	397	
3	469651		10	67093		17	397	
4	469651		11	67093		18	397	
5	469651		12	67093		19	397	
6	469651		13	67093	13 13	20	397	
7	469651	7	14	397				397
8	67093		15	397				

3757208/8

Debugging: 95% of Program Development

Programming. A process of finding and fixing mistakes.

- Compiler error messages help locate **syntax** errors.
- Run program to find **semantic** and **performance** errors.

```
public class Factors {
    public static void main(String[] args) {
        long N = Long.parseLong(args[0])
        for (i = 0; i < N; i++) {
            while (N % i == 0)
                System.out.print(i + " ")
                N = N / i
        }
    }
}
```

check if i is a factor →

← as long as i is a factor, divide it out

this program has many bugs!

Debugging: Syntax Errors

Syntax error. Illegal Java program.

- Compiler error messages help locate problem.
- Goal: no errors and a file named `Factors.class`.

```
public class Factors {
    public static void main(String[] args) {
        long N = Long.parseLong(args[0])
        for (i = 0; i < N; i++) {
            while (N % i == 0)
                System.out.print(i + " ")
                N = N / i
        }
    }
}
```

```
% javac Factors.java
Factors.java:6: ';' expected
        for (i = 2; i < N; i++)
            ^
1 error ← the first error
```

Debugging: Syntax Errors

Syntax error. Illegal Java program.

- Compiler error messages help locate problem.
- Goal: no errors and a file named `Factors.class`.

```
public class Factors {
    public static void main(String[] args) {
        long N = Long.parseLong(args[0]);
        for (int i = 0; i < N; i++) {
            while (N % i == 0)
                System.out.print(i + " ");
            N = N / i;
        }
    }
}
```

need to declare variable i

need terminating semicolons

syntax (compile-time) errors



Debugging: Semantic Errors

Semantic error. Legal but wrong Java program.

- Run program to identify problem.
- Add print statements if needed to produce trace.

```
public class Factors {  
    public static void main(String[] args) {  
        long N = Long.parseLong(args[0]);  
        for (int i = 0; i < N; i++) {  
            while (N % i == 0)  
                System.out.print(i + " ");  
            N = N / i;  
        }  
    }  
}
```

```
% javac Factors.java  
% java Factors ← oops, no argument  
Exception in thread "main"  
java.lang.ArrayIndexOutOfBoundsException: 0  
    at Factors.main(Factors.java:5)
```

Debugging: Semantic Errors

Semantic error. Legal but wrong Java program.

- Run program to identify problem.
- Add print statements if needed to produce trace.

```
public class Factors {  
    public static void main(String[] args) {  
        long N = Long.parseLong(args[0]);  
        for (int i = 0; i < N; i++) {  
            while (N % i == 0)  
                System.out.print(i + " ");  
            N = N / i;  
        }  
    }  
}
```

need to start at 2
because 0 and 1
cannot be factors

```
% javac Factors.java  
% java Factors 98  
Exception in thread "main"  
java.lang.ArithmeticException: / by zero  
    at Factors.main(Factors.java:8)
```

Debugging: Semantic Errors

Semantic error. Legal but wrong Java program.

- Run program to identify problem.
- Add print statements if needed to produce trace.

```
public class Factors {  
    public static void main(String[] args) {  
        long N = Long.parseLong(args[0]);  
        for (int i = 2; i < N; i++) {  
            while (N % i == 0)  
                System.out.print(i + " ");  
            N = N / i;  
        }  
    }  
}
```

indents do not
imply braces

```
% javac Factors.java  
% java Factors 98  
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2  
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2  
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 ...
```

infinite loop!

Debugging: The Beat Goes On

Success. Program factors $98 = 2 \times 7^2$.

- But that doesn't mean it works for all inputs.
- Add trace to find and fix (minor) problems.

```
public class Factors {  
    public static void main(String[] args) {  
        long N = Long.parseLong(args[0]);  
        for (int i = 2; i < N; i++) {  
            while (N % i == 0) {  
                System.out.print(i + " ");  
                N = N / i;  
            }  
        }  
    }  
}
```

```
% java Factors 98
```

```
2 7 %
```

← need newline

```
% java Factors 5
```

← ??? no output

```
% java Factors 6
```

```
2 %
```

← ??? missing the 3



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```
public class Factors {  
    public static void main(String[] args) {  
        long N = Long.parseLong(args[0]);  
        for (int i = 2; i < N; i++) {  
            while (N % i == 0) {  
                System.out.println(i + " ");  
                N = N / i;  
            }  
            System.out.println("TRACE: " + i + " " + N);  
        }  
    }  
}
```


```
% java Factors 5  
TRACE 2 5  
TRACE 3 5  
TRACE 4 5  
  
% java Factors 6  
2  
TRACE 2 3
```

Aha!
Print out N
after for loop
(if it is not 1)

Debugging: Success?

Success. Program seems to work.

```
public class Factors {
    public static void main(String[] args) {
        long N = Long.parseLong(args[0]);
        for (int i = 2; i < N; i++) {
            while (N % i == 0) {
                System.out.print(i + " ");
                N = N / i;
            }
        }
        if (N > 1) System.out.println(N);
        else      System.out.println();
    }
}
```

 "corner case"

```
% java Factors 5
5

% java Factors 6
2 3

% java Factors 98
2 7 7

% java Factors 3757208
2 2 2 7 13 13 397
```

Debugging: Performance Error

Performance error. Correct program, but too slow.

```
public class Factors {
    public static void main(String[] args) {
        long N = Long.parseLong(args[0]);
        for (int i = 2; i < N; i++) {
            while (N % i == 0) {
                System.out.print(i + " ");
                N = N / i;
            }
        }
        if (N > 1) System.out.println(N);
        else      System.out.println();
    }
}
```

```
% java Factors 11111111
11 73 11 137

% java Factors 11111111111
21649 51329

% java Factors 111111111111111
11 239 4649 909091

% java Factors 111111111111111111
2071723
```

← very long wait
(with a surprise ending)

Debugging: Performance Error

Performance error. Correct program, but too slow.

Solution. Improve or change underlying algorithm.

fixes performance error:
if N has a factor, it has one
less than or equal to its square root

```
public class Factors {
    public static void main(String[] args) {
        long N = Long.parseLong(args[0]);
        for (int i = 2; i <= N/i; i++) {
            while (N % i == 0) {
                System.out.print(i + " ");
                N = N / i;
            }
        }
        if (N > 1) System.out.println(N);
        else      System.out.println();
    }
}
```

```
% java Factors 11111111
11 73 11 137

% java Factors 11111111111
21649 51329

% java Factors 11111111111111
11 239 4649 909091

% java Factors 1111111111111111
2071723 5363222357
```



Program Development: Analysis

Q. How large an integer can I factor?

```

% java Factors 3757208
2 2 2 7 13 13 397

% java Factors 9201111169755555703
9201111169755555703

```

after a few minutes of computing...

largest factor →

digits	($i \leq N$)	($i*i \leq N$)
3	instant	instant
6	0.15 seconds	instant
9	77 seconds	instant
12	21 hours †	0.16 seconds
15	2.4 years †	2.7 seconds
18	2.4 millennia †	92 seconds

† estimated

Note. Can't break RSA this way (experts are still trying).

Debugging

Programming. A process of finding and fixing mistakes.

1. Create the program.
2. Compile it.
Compiler says: That's not a legal program.
Back to step 1 to fix syntax errors.
3. Execute it.
Result is bizarrely (or subtly) wrong.
Back to step 1 to fix semantic errors.
4. Enjoy the satisfaction of a working program!
5. Too slow? Back to step 1 to try a different algorithm.



U.S.S. Grace Murray Hopper



Extra Slides



Oblivious Sorting

Sort. Read in 3 integers and rearrange them in ascending order.

```
public class Sort3 {  
    public static void main(String[] args) {  
  
        int a = Integer.parseInt(args[0]);  
        int b = Integer.parseInt(args[1]);  
        int c = Integer.parseInt(args[2]);  
  
        if (b > c) { int t = b; b = c; c = t; }  
        if (a > b) { int t = a; a = b; b = t; }  
        if (b > c) { int t = b; b = c; c = t; }  
  
        System.out.println(a + " " + b + " " + c);  
    }  
}
```

read in 3 integers
from command-line

swap b and c
swap a and b
swap b and c

```
% java Sort3 9 8 7  
7 8 9
```

```
% java Sort3 2 1 7  
1 2 7
```

Puzzle 1. Sort 4 integers with 5 compare-exchanges.

Puzzle 2. Sort 6 integers with 12.



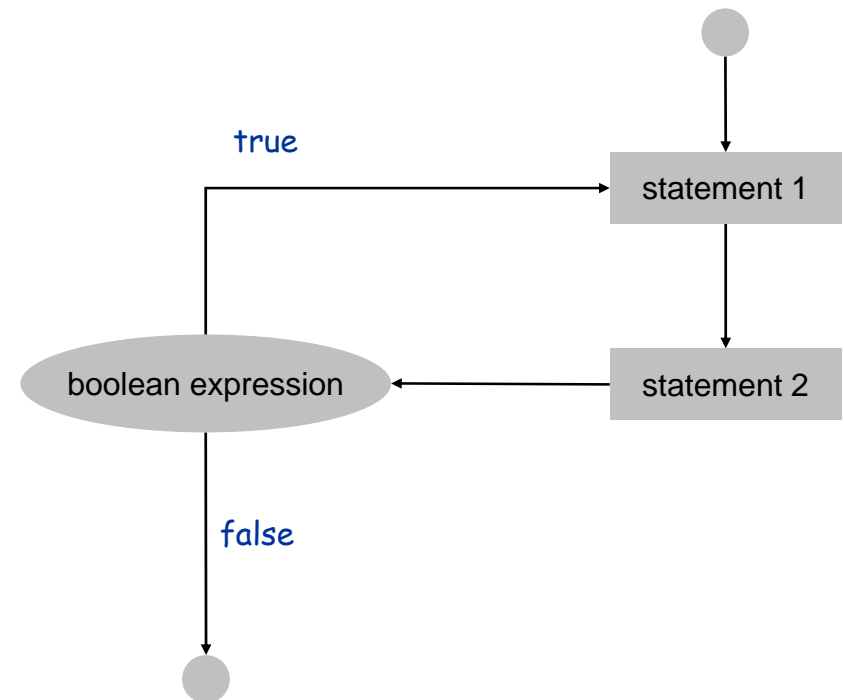
Do-While Loop

The **do-while** loop. A less common repetition structure.

- Execute sequence of statements.
- Check loop-continuation condition.
- Repeat.

```
do {  
    statement 1;  
    statement 2;  
} while (boolean expression);
```

do-while loop syntax



Do-While Loop

Ex. Find a point (x, y) that is uniformly distributed in unit disc.

- Pick a random point in unit square.
- Check if point is also in unit disc.
- Repeat.

```
do {  
  x = 2.0 * Math.random() - 1.0;  
  y = 2.0 * Math.random() - 1.0;  
} while (x*x + y*y > 1.0);
```

between -1 and 1

